Reconfigurable cooperative control for extremum seeking

Arthur Kahn (ONERA), Julien Marzat (ONERA),
Hélène Piet Lahanier (ONERA), Michel Kieffer (L2S)

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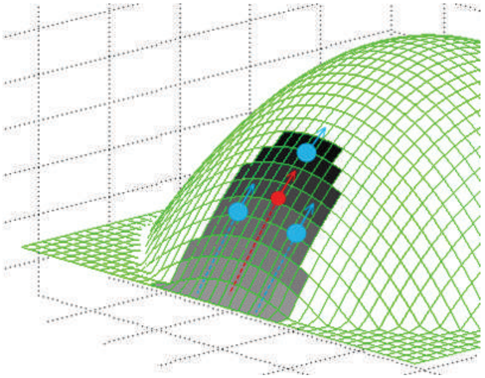
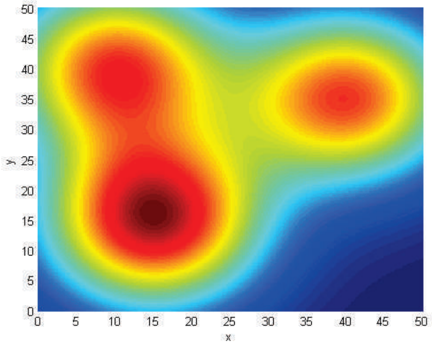
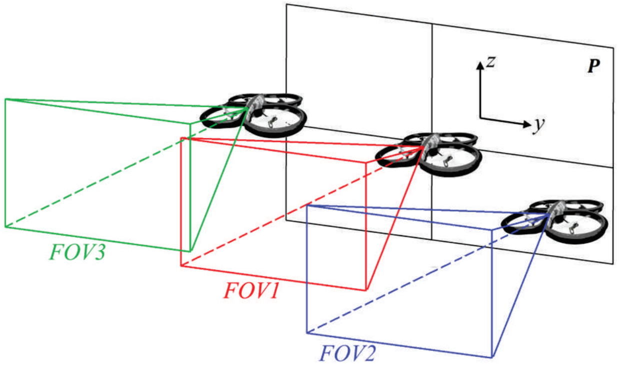
Context

Source location and surveillance missions

- Forest fire source location
- Chemical or gas leaks
- Surveillance of large areas or search and rescue







Interest for multi-vehicle systems (MVS)

- Mission repartition
- Robustness to faults or agent loss

Field maximization with a MVS

Goal

$\rightarrow$ Find the global maximum of an initially unknown spatial field

Means

$\rightarrow$ Multi-vehicle system (MVS)

$\rightarrow$ Each vehicle measures the field value at its position

Constraints

$\rightarrow$ Accurately locate field maximum

$\rightarrow$ Take into account vehicle dynamics

$\rightarrow$ Avoid collisions between vehicles

$\rightarrow$ Limit the number of measurements

Outline

Assumptions

Consider some unknown, continuous, and time-invariant scalar field

$$\phi : \mathbf{x} \in \mathscr{D} \subset \mathbb{R}^2 \to \phi(\mathbf{x}) \in \mathbb{R}$$

to be maximized using N identical mobile agents with dynamics

$$M\ddot{\mathbf{x}}_i + C(\mathbf{x}_i, \dot{\mathbf{x}}_i)\dot{\mathbf{x}}_i = \mathbf{u}_i,$$

and measurement equation at $\mathbf{x}_i$

$$y(\mathbf{x}_i) = \phi(\mathbf{x}_i) + w_i(\eta_i),$$

w_i measurement noise and η_i the i-th sensor state

- $\eta_i = 0$ nominal sensor
- $\eta_i = 1$ faulty sensor (bias or modified variance)

Problem statement

- N identical vehicles with lossless synchronized communication
- Communication radius R defines agent i neighbourhood

$$\mathscr{N}_i(t) = \{j \mid \|\mathbf{x}_i(t) - \mathbf{x}_j(t)\| \leqslant R\}.$$

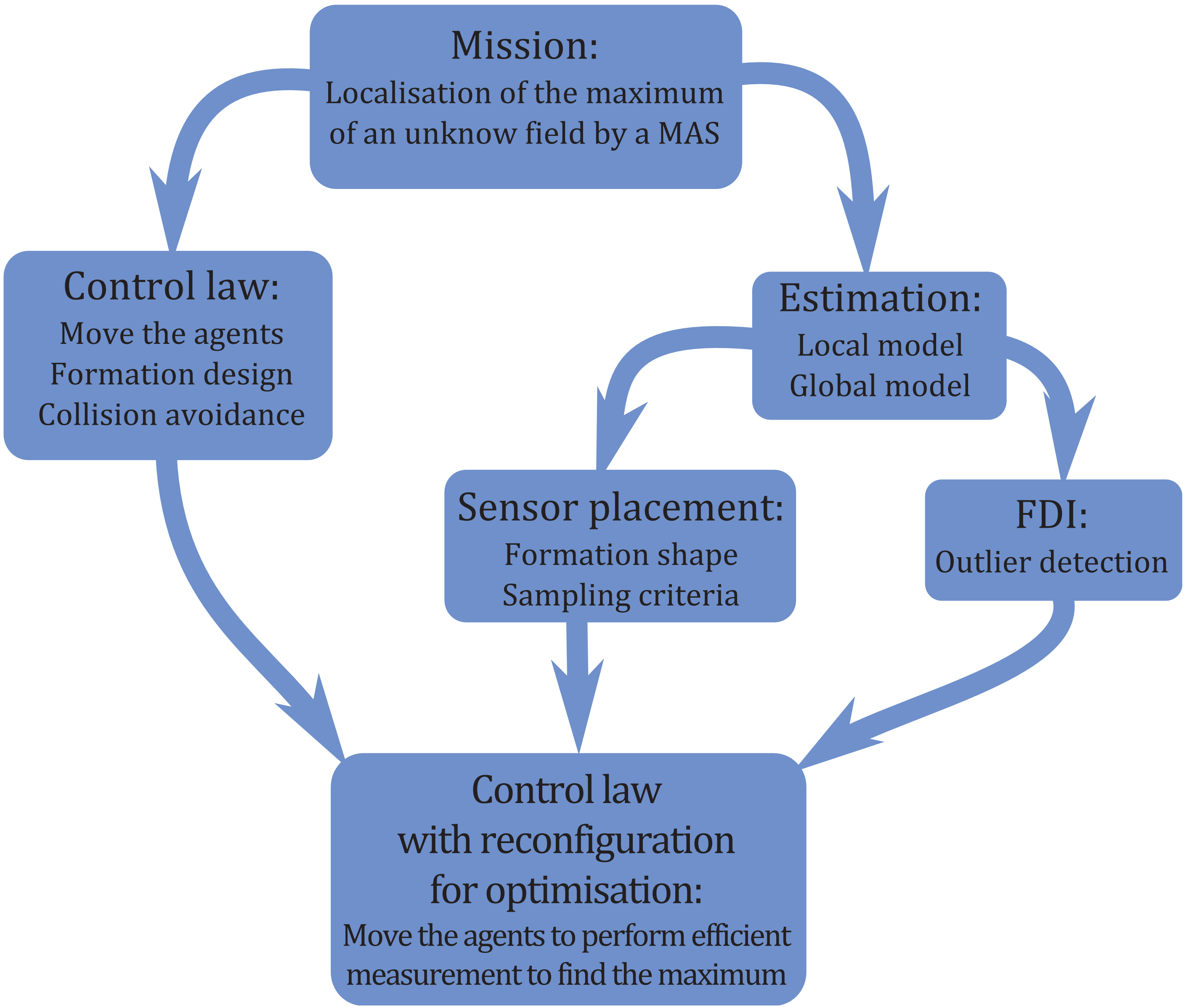
- Available information at time t_k for agent i

$$\mathscr{S}_i(t_k) = \bigcup_{\ell=0}^{k} \{[y_j(t_\ell), \mathbf{x}_j(t_\ell)] \mid j \in \mathscr{N}_i(t_\ell) \cap \mathscr{M}(t_\ell)\}.$$

- Define a strategy to find efficiently (time, measurements)

$$\mathbf{x}_M = \arg\max_{\mathbf{x}\in D} \{\phi(\mathbf{x})\}$$

Topics addressed



Proposed solutions

1 Define iteratively vehicle sampling positions

2 Model computation from measurements

3 Move vehicles with collision avoidance

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Proposed solutions

Local approach

1 Define iteratively vehicle sampling positions

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Optimal sensor placement

Proposed solutions

	Local approach
1 Define iteratively vehicle sampling positions	Optimal sensor placement
2 Model computation from measurements	Local linear model
3 Move vehicles with collision avoidance	

Proposed solutions

	Local approach
1 Define iteratively vehicle sampling positions	Optimal sensor placement
2 Model computation from measurements	Local linear model
3 Move vehicles with collision avoidance	Formation control

Proposed solutions

Global approach

1 Define iteratively vehicle sampling positions

Constrained sampling criterion

2 Model computation from measurements

3 Move vehicles with collision avoidance

Proposed solutions

	Global approach
1 Define iteratively vehicle sampling positions	Constrained sampling criterion
2 Model computation from measurements	Kriging model of the field
3 Move vehicles with collision avoidance	

Proposed solutions

	Global approach
1 Define iteratively vehicle sampling positions	Constrained sampling criterion
2 Model computation from measurements	Kriging model of the field
3 Move vehicles with collision avoidance	Spread the vehicles in the area

Section 2

Local approach

Local approach

"Gradient climbing" algorithm (*Ögren 2004, Cortes 2009*)

1. Vehicles are kept in a close formation
2. Vehicles measure the field value at their positions and broadcast
3. Cooperative gradient estimation from measurements
4. Computation of formation motion along gradient direction

Contributions

- Cooperative weighted least-square estimation with local model
- Outlier detection: adaptive threshold related to cooperative estimation model
- Optimal sensor placement with faulty sensors (Fisher information matrix)
- Fleet control: vehicle formation motion and reconfiguration

Field modeling

Locally, spatial field ϕ can be written

$$\phi_i(\mathbf{x}) = \phi\left(\widehat{\mathbf{x}}_i^k\right) + \left(\mathbf{x} - \widehat{\mathbf{x}}_i^k\right)^{\mathsf{T}} \nabla\phi\left(\widehat{\mathbf{x}}_i^k\right) + \frac{1}{2}\left(\mathbf{x} - \widehat{\mathbf{x}}_i^k\right)^{\mathsf{T}} \nabla^2\phi(\chi_i)\left(\mathbf{x} - \widehat{\mathbf{x}}_i^k\right).$$

Parameter vector $\alpha_i^k = \begin{pmatrix} \phi\left(\widehat{\mathbf{x}}_i^k\right) \\ \nabla\phi\left(\widehat{\mathbf{x}}_i^k\right) \end{pmatrix}$ to be estimated

Local linear model

$$\overline{\phi}_i(\mathbf{x}) = \phi\left(\widehat{\mathbf{x}}_i^k\right) + \left(\mathbf{x} - \widehat{\mathbf{x}}_i^k\right)^{\mathsf{T}} \nabla\phi\left(\widehat{\mathbf{x}}_i^k\right),$$

with modeling error $e_i(\mathbf{x}) = \phi_i(\mathbf{x}) - \overline{\phi}_i(\mathbf{x})$

Weighted least-squares

Measurement of vehicle j

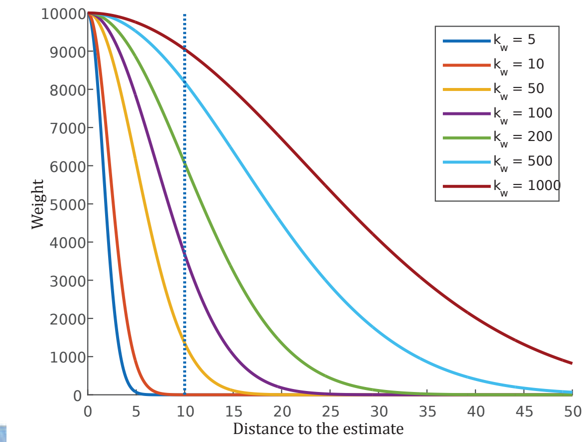
$$y_j(t_k) = \begin{pmatrix} 1 & (\mathbf{x}_j(t_k) - \widehat{\mathbf{x}}_i^k)^{\mathsf{T}} \end{pmatrix} \alpha_i^k + e_i(\mathbf{x}_j(t_k)) + n_j(t_k).$$

Vehicle i collects all measurements from $\mathcal{N}_i(t_k)$

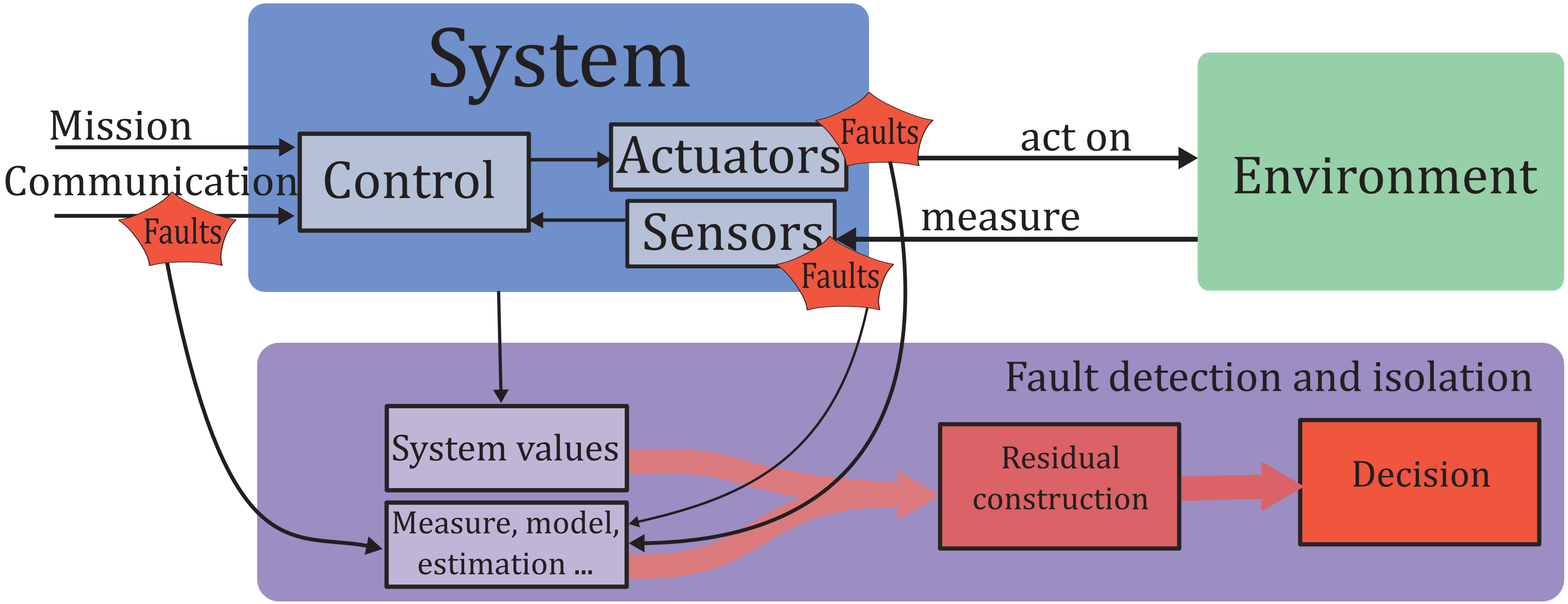
$$\mathbf{y}_{i,k} = \overline{\mathbf{R}}_{i,k}\alpha_i^k + \mathbf{n}_{i,k} + \mathbf{e}_{i,k}$$

Weight matrix $\mathbf{W}_{i,k} =$
$$\operatorname{diag}\left(\sigma_{\eta_1(t_k)}^{-2} \exp\left(\frac{-\|\mathbf{x}_1(t_k) - \widehat{\mathbf{x}}_i^k\|_2^2}{k_w}\right), \dots, \sigma_{\eta_N(t_k)}^{-2} \exp\left(\frac{-\|\mathbf{x}_N(t_k) - \widehat{\mathbf{x}}_i^k\|_2^2}{k_w}\right)\right)$$

$$\widehat{\alpha}_i^k = \left(\overline{\mathbf{R}}_{i,k}^{\mathsf{T}} \mathbf{W}_{i,k} \overline{\mathbf{R}}_{i,k}\right)^{-1} \overline{\mathbf{R}}_{i,k}^{\mathsf{T}} \mathbf{W}_{i,k} \mathbf{y}_{i,k}$$



Model-based fault detection scheme



Problem characteristics

- Local model shared by vehicles
- Spatially-varying modeling error

Fault detection

Faulty sensor of vehicle i : $y_i = \phi(\mathbf{x}_i) + w_i(\eta_i) + d$

Detection residual $r_i = \hat{\phi}_i(\mathbf{x}_i) - y_i$

Adaptive threshold for residual analysis

$$|r_i| < k_{\mathrm{FDI}} \sqrt{\sigma_0^2 \left(1 + \mathbf{h}_i \mathbf{h}_i^T - 2\mathbf{h}_i[i]\right) + \mathbf{h}_i^T \mathbf{U}_i \mathbf{h}_i}$$

Takes into account measurement noise, sensor locations and modeling error

For fault isolation:

- For each vehicle, bank of N residuals r_{ij} excluding the j-th measurement
- Consensus between vehicles to identify the faulty sensors

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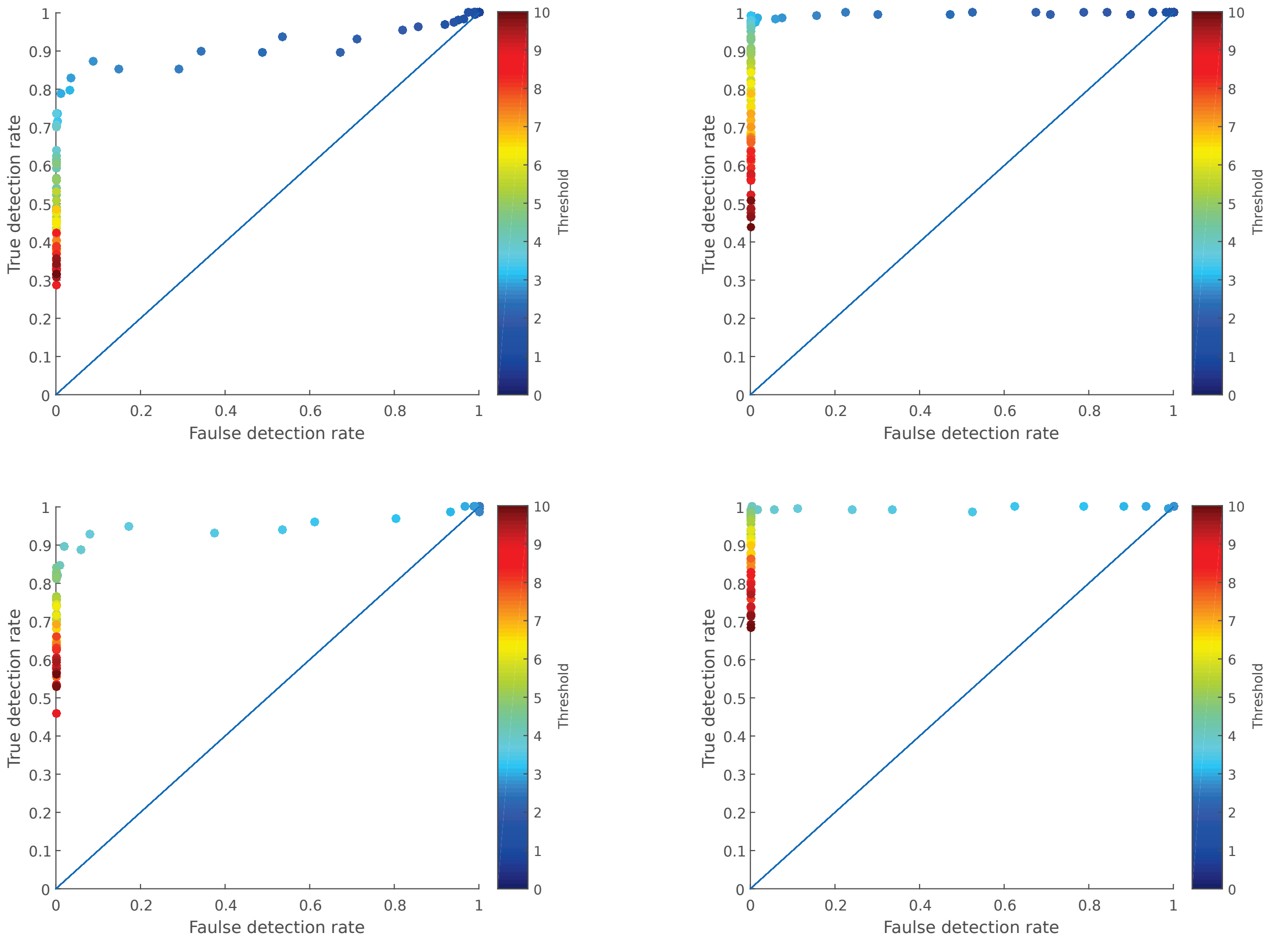
Fault detection results

$d = 3$ $d = 5$

$N = 7$

$N = 15$



Optimal sensor placement

Find sensor locations (and associated formation shapes) that

- minimise estimate variance and modeling error influence
- take into account different sensor variances (faults)

Minimise a function of estimation error covariance matrix

$$\widehat{\boldsymbol{\Sigma}}_{\alpha_i^{k+1}} = \left(\overline{\mathbf{R}}^{\mathsf{T}}_{i,k+1}\mathbf{W}_{i,k+1}\overline{\mathbf{R}}_{i,k+1}\right)^{-1}$$

under collision avoidance constraint $\|\mathbf{x}_i - \mathbf{x}_j\|_2^2 \geqslant R^2_{\text{safety}},\ \forall\{i,j\}, j > i$

Several optimal design criteria *(Walter & Pronzato 1987)*

Optimal sensor placement

T-optimal solution

$$\left(\mathbf{x}_1\left(t_{k+1}\right)\ldots\mathbf{x}_N\left(t_{k+1}\right)\right) = \arg\max_{\left(\mathbf{x}_1,\ldots,\mathbf{x}_N\right)} \operatorname{tr}\left(\overline{\mathbf{R}}^{\mathsf{T}}_{i,k+1}\mathbf{W}_{i,k+1}\overline{\mathbf{R}}_{i,k+1}\right)$$

$$\text{s.t. } \|\mathbf{x}_i - \mathbf{x}_j\|_2^2 \geqslant R^2_{\text{safety}}, \ \forall\{i,j\}, j > i.$$

Optimal sensor placement

T-optimal solution

$$\left(\mathbf{x}_1(t_{k+1})\ldots\mathbf{x}_N(t_{k+1})\right) = \arg\max_{(\mathbf{x}_1,\ldots,\mathbf{x}_N)} \operatorname{tr}\left(\overline{\mathbf{R}}^{\mathrm{T}}_{i,k+1}\mathbf{W}_{i,k+1}\overline{\mathbf{R}}_{i,k+1}\right)$$

$$\text{s.t. } \|\mathbf{x}_i - \mathbf{x}_j\|_2^2 \geqslant R^2_{\text{safety}},\ \forall\{i,j\}, j > i.$$

Lagrangian $\mathscr{L} = \operatorname{tr}\left(\overline{\mathbf{R}}^{\mathrm{T}}_{i,k+1}\mathbf{W}_{i,k+1}\overline{\mathbf{R}}_{i,k+1}\right) + \sum_{j>i}\mu_{ij}\left(\|\mathbf{x}_i - \mathbf{x}_j\|_2^2 - R^2_{\text{safety}}\right)$

Two solutions for $\mu_{ij} = 0$ (inactive constraints),

$$\mathbf{x}_i(t_{k+1}) = \widehat{\mathbf{x}}_i^{k+1} \qquad\qquad \left\|\mathbf{x}_i(t_{k+1}) - \widehat{\mathbf{x}}_i^{k+1}\right\|_2^2 = k_w - 1$$

Optimal sensor placement

D-optimal solution

$$\left(\mathbf{x}_1\left(t_{k+1}\right)\ldots\mathbf{x}_N\left(t_{k+1}\right)\right) = \arg\max_{(\mathbf{x}_1,\ldots,\mathbf{x}_N)} \det\left(\overline{\mathbf{R}}_{i,k+1}^{\mathrm{T}}\mathbf{W}_{i,k+1}\overline{\mathbf{R}}_{i,k+1}\right)$$

$$\text{s.t. } \|\mathbf{x}_i - \mathbf{x}_j\|_2^2 \geqslant R_{\text{safety}}^2,\ \forall\{i,j\}, j > i.$$

Optimal sensor placement

D-optimal solution

$$\left(\mathbf{x}_1\left(t_{k+1}\right)\ldots\mathbf{x}_N\left(t_{k+1}\right)\right)=\arg\max_{\left(\mathbf{x}_1,\ldots,\mathbf{x}_N\right)}\det\left(\overline{\mathbf{R}}^{\mathrm{T}}_{i,k+1}\mathbf{W}_{i,k+1}\overline{\mathbf{R}}_{i,k+1}\right)$$
$$\text{s.t. } \|\mathbf{x}_i-\mathbf{x}_j\|_2^2\geqslant R^2_{\text{safety}},\ \forall\{i,j\},j>i.$$

Lagrangian $\mathscr{L}=\det\left(\overline{\mathbf{R}}^{\mathrm{T}}_{i,k+1}\mathbf{W}_{i,k+1}\overline{\mathbf{R}}_{i,k+1}\right)+\sum_{j>i}\mu_{ij}\left(\|\mathbf{x}_i-\mathbf{x}_j\|_2^2-R^2_{\text{safety}}\right)$

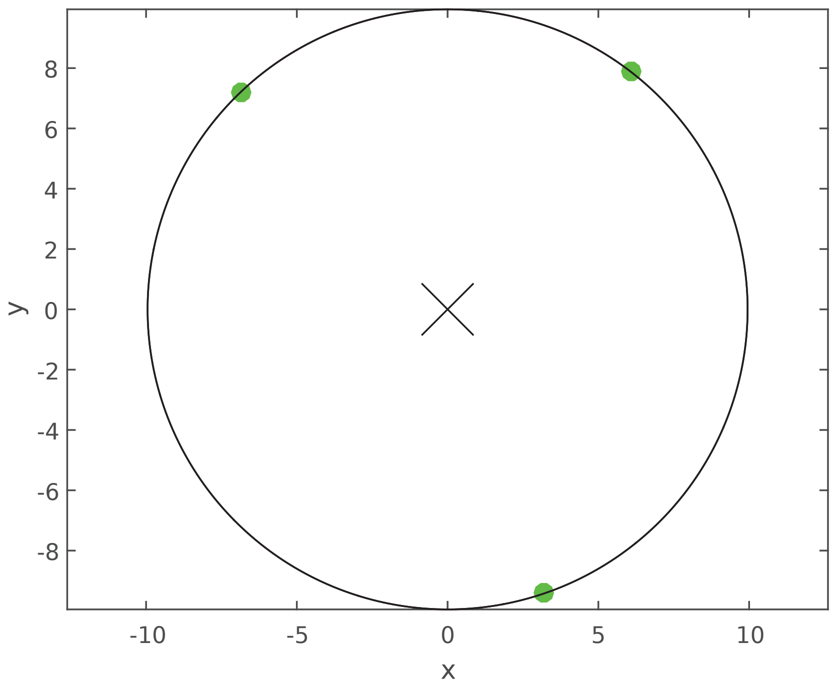
Two solutions for $\mu_{ij}=0$ (inactive constraints),

$$\mathbf{x}_i(t_{k+1})=\widehat{\mathbf{x}}_i^{k+1} \qquad \left\|\mathbf{x}_i\left(t_{k+1}\right)-\widehat{\mathbf{x}}_i^{k+1}\right\|_2^2=\frac{2k_{\mathrm{w}}}{3}$$

Numerical solutions

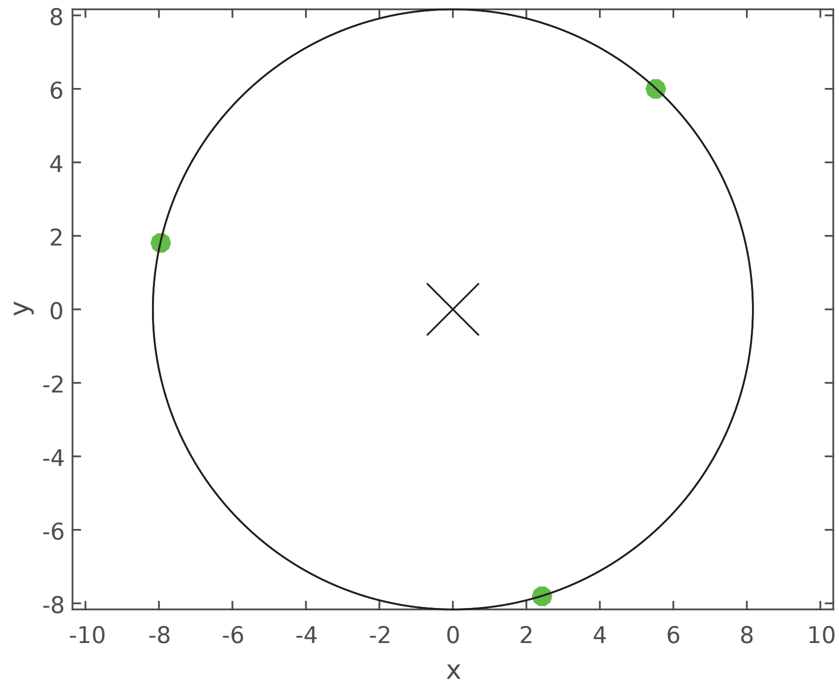
$N = 3$ agents, no faulty agent

T-optimal



$$\text{radius} = \sqrt{k_w - 1}$$

D-optimal

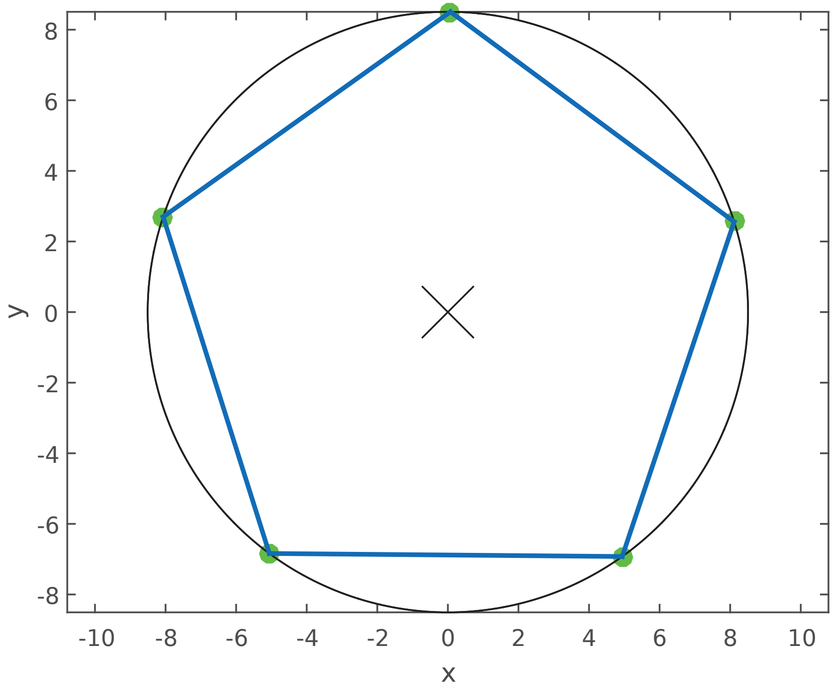


$$\text{radius} = \sqrt{\frac{2k_w}{3}}$$

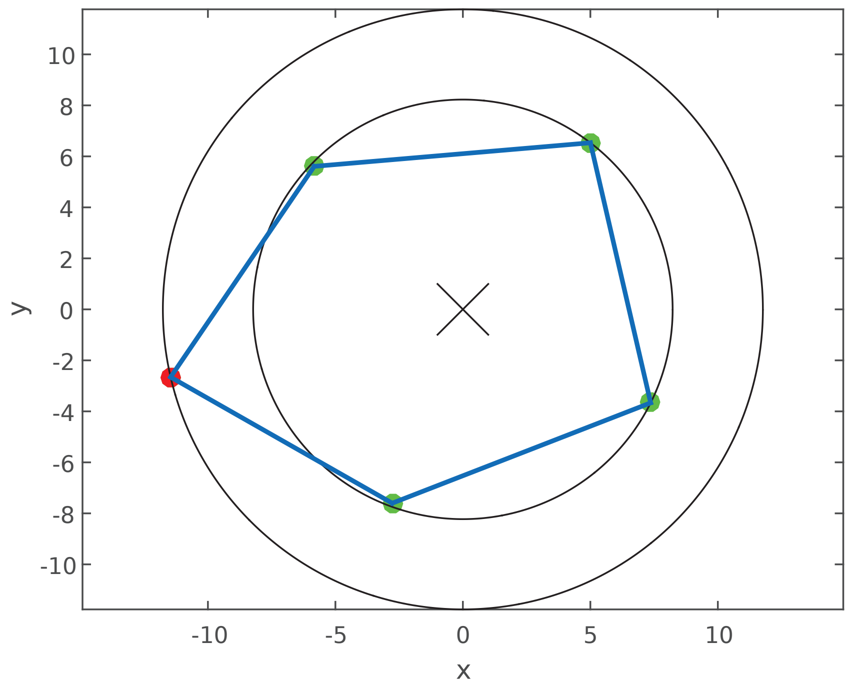
Numerical solutions

$N = 5$ agents, D-optimal placement

No fault



1 faulty agent



Numerical solutions

Conclusion on T-optimal and D-optimal sensor placement

- All vehicles should be located on a circle with inactive constraints
- A faulty agent is placed further from the fleet, due to estimation weight

Sensor placement to minimize modeling error

- Be as close as possible to estimation position

Formation characteristics

- T-optimal $\rightarrow$ concentric circles
- D-optimal $\rightarrow$ *compact* formation around estimation position with active constraints

A. Kahn, J. Marzat, H. Piet-Lahanier, M. Kieffer, Cooperative estimation with outlier detection and fleet reconfiguration for multi-agent systems, IFAC Workshop on Multi-Vehicule Systems 2015

Cooperative guidance law

- Manage vehicle motions to respect sensor placement
- Locate field maximum

A virtual point $\widehat{\mathbf{x}}^k$ is used in a two-layer control law

- High-level control
  - Move the virtual point to track the field maximum
- Low-level control
  - Keep the agents in formation around the virtual point
  - Avoid collisions between vehicles

High-level control

Gradient climbing of estimation position $\widehat{\mathbf{x}}^k$

$$\widehat{\mathbf{x}}^{k+1} = \widehat{\mathbf{x}}^k + \lambda^k \widehat{\nabla\phi}\left(\widehat{\mathbf{x}}^k\right) / \left\|\widehat{\nabla\phi}\left(\widehat{\mathbf{x}}^k\right)\right\|_2 .$$

$\widehat{\mathbf{x}}^k$ can be proven to converge to maximum for concave fields

Decentralized computation of estimation position is possible with incomplete communication graph

J. Marzat, A. Kahn, H. Piet-Lahanier Cooperative guidance of Lego Mindstorms NXT mobile robots, 11th International Conference on Informatics in Control, Automation and Robotics, Vienne Autriche, 2014

Experiment

Low-level control

Vehicle dynamics

$$M\ddot{\mathbf{x}}_i(t) + C\left(\mathbf{x}_i(t), \dot{\mathbf{x}}_i(t)\right)\dot{\mathbf{x}}_i(t) = \mathbf{u}_i(t)$$

Proposed control law (similar to *Cheah, 2009*)

$$\begin{aligned}
\mathbf{u}_i(t) =& M\ddot{\widehat{\mathbf{x}}}_i(t) + C(\mathbf{x}_i(t), \dot{\mathbf{x}}_i(t))\dot{\mathbf{x}}_i(t) - k_1\left(\dot{\mathbf{x}}_i(t) - \dot{\widehat{\mathbf{x}}}_i(t)\right) \\
&+ 2k_2\sum_{j=1}^{N}(\mathbf{x}_i(t) - \mathbf{x}_j(t))\exp\left(-\frac{(\mathbf{x}_i(t) - \mathbf{x}_j(t))^T(\mathbf{x}_i(t) - \mathbf{x}_j(t))}{q}\right) \\
&- k_3^i(\eta_i, t)(\mathbf{x}_i(t) - \widehat{\mathbf{x}}_i(t)),
\end{aligned}$$

Control stability

Candidate Lyapunov function $V(\mathbf{X}(t))$

$$V(\mathbf{X}(t)) = \frac{1}{2}\sum_{i=1}^{N}\Bigg[(\dot{\mathbf{x}}_i(t) - \dot{\widehat{\mathbf{x}}}(t))^T M(\dot{\mathbf{x}}_i(t) - \dot{\widehat{\mathbf{x}}}(t))$$
$$+ (\mathbf{x}_i(t) - \widehat{\mathbf{x}}(t))^T k_3^i(\mathbf{x}_i(t) - \widehat{\mathbf{x}}(t))$$
$$+ k_2\sum_{j=1}^{N}\exp\left(-\frac{(\mathbf{x}_i(t) - \mathbf{x}_j(t))^T(\mathbf{x}_i(t) - \mathbf{x}_j(t))}{q}\right)\Bigg]$$

Control stability

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$$V(\mathbf{X}(t)) = \frac{1}{2}\sum_{i=1}^{N}\Bigg[(\dot{\mathbf{x}}_i(t) - \dot{\widehat{\mathbf{x}}}(t))^T M(\dot{\mathbf{x}}_i(t) - \dot{\widehat{\mathbf{x}}}(t)) + (\mathbf{x}_i(t) - \widehat{\mathbf{x}}(t))^T k_3^i(\mathbf{x}_i(t) - \widehat{\mathbf{x}}(t)) + k_2\sum_{j=1}^{N}\exp\left(-\frac{(\mathbf{x}_i(t) - \mathbf{x}_j(t))^T(\mathbf{x}_i(t) - \mathbf{x}_j(t))}{q}\right)\Bigg]$$

Speed control term

Control stability

Candidate Lyapunov function $V(\mathbf{X}(t))$

$$V(\mathbf{X}(t)) = \frac{1}{2}\sum_{i=1}^{N}\Big[(\dot{\mathbf{x}}_i(t) - \dot{\widehat{\mathbf{x}}}(t))^T M(\dot{\mathbf{x}}_i(t) - \dot{\widehat{\mathbf{x}}}(t)) + (\mathbf{x}_i(t) - \widehat{\mathbf{x}}(t))^T k_3^i(\mathbf{x}_i(t) - \widehat{\mathbf{x}}(t)) + k_2 \sum_{j=1}^{N} \exp\left(-\frac{(\mathbf{x}_i(t) - \mathbf{x}_j(t))^T(\mathbf{x}_i(t) - \mathbf{x}_j(t))}{q}\right)\Big]$$

Position control term

Control stability

Candidate Lyapunov function $V(\mathbf{X}(t))$

$$V(\mathbf{X}(t)) = \frac{1}{2}\sum_{i=1}^{N}\Bigg[(\dot{\mathbf{x}}_i(t) - \dot{\widehat{\mathbf{x}}}(t))^T M(\dot{\mathbf{x}}_i(t) - \dot{\widehat{\mathbf{x}}}(t)) + (\mathbf{x}_i(t) - \widehat{\mathbf{x}}(t))^T k_3^i(\mathbf{x}_i(t) - \widehat{\mathbf{x}}(t)) + k_2\sum_{j=1}^{N}\exp\left(-\frac{(\mathbf{x}_i(t) - \mathbf{x}_j(t))^T(\mathbf{x}_i(t) - \mathbf{x}_j(t))}{q}\right)\Bigg]$$

Collision avoidance term

Control stability

Candidate Lyapunov function $V(\mathbf{X}(t))$

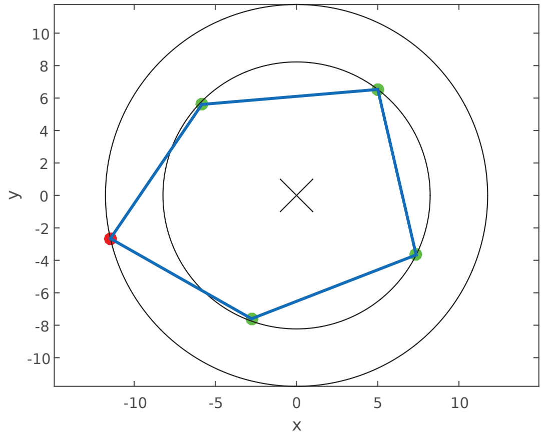
$$V(\mathbf{X}(t)) = \frac{1}{2}\sum_{i=1}^{N}\Bigg[(\dot{\mathbf{x}}_i(t) - \dot{\widehat{\mathbf{x}}}(t))^T M(\dot{\mathbf{x}}_i(t) - \dot{\widehat{\mathbf{x}}}(t)) + (\mathbf{x}_i(t) - \widehat{\mathbf{x}}(t))^T k_3^i(\mathbf{x}_i(t) - \widehat{\mathbf{x}}(t)) + k_2\sum_{j=1}^{N}\exp\left(-\frac{(\mathbf{x}_i(t) - \mathbf{x}_j(t))^T(\mathbf{x}_i(t) - \mathbf{x}_j(t))}{q}\right)\Bigg]$$

Control law can be proven to be Lyapunov stable

A. Kahn, J. Marzat, H. Piet-Lahanier, M. Kieffer, Cooperative estimation with outlier detection and fleet reconfiguration for multi-agent systems, IFAC Workshop on Multi-Vehicule Systems, Gênes Italie, 2015

Reconfiguration

Optimal sensor placement $\rightarrow$ desired formation shape

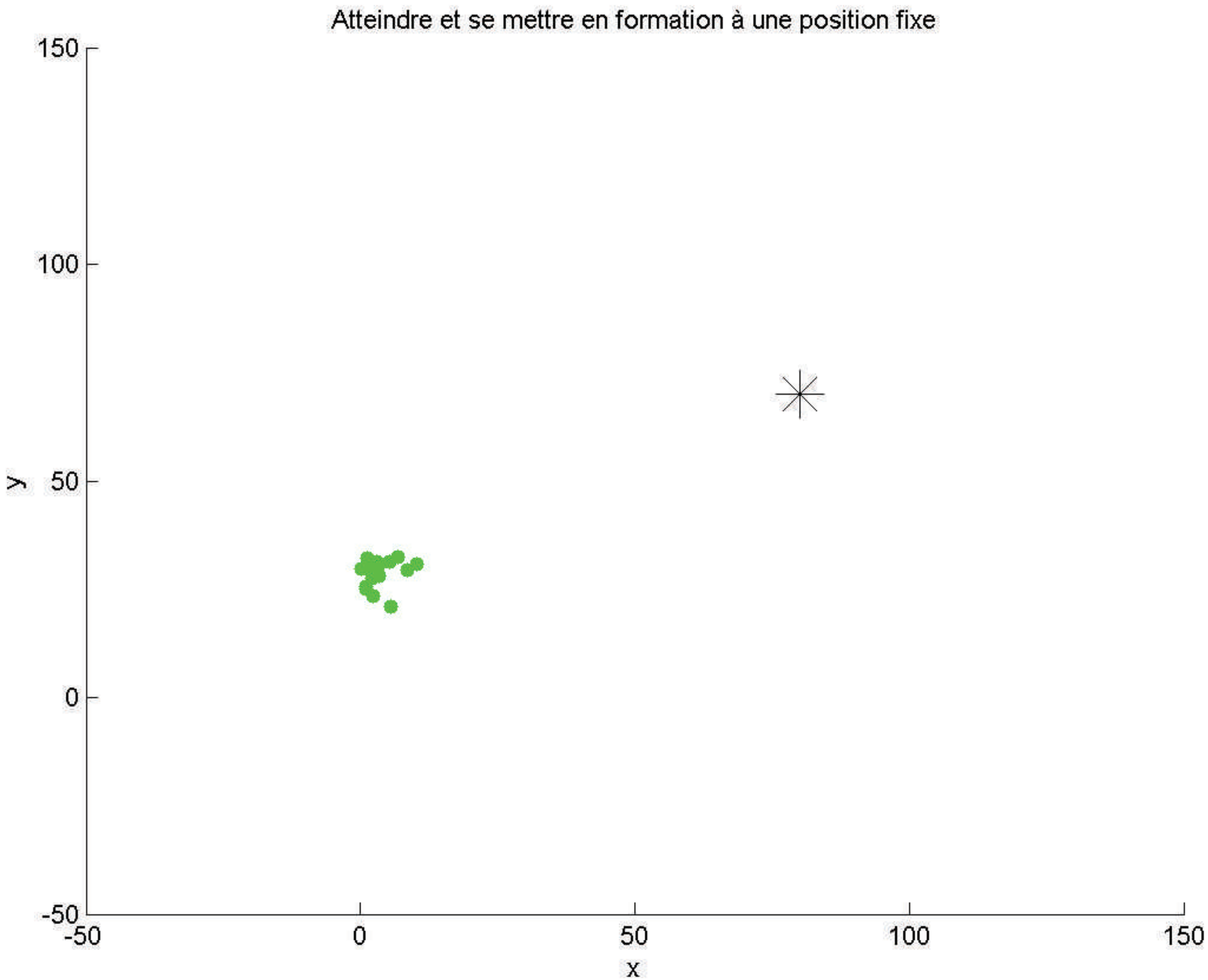


Faulty agent i $\rightarrow$ modified control law

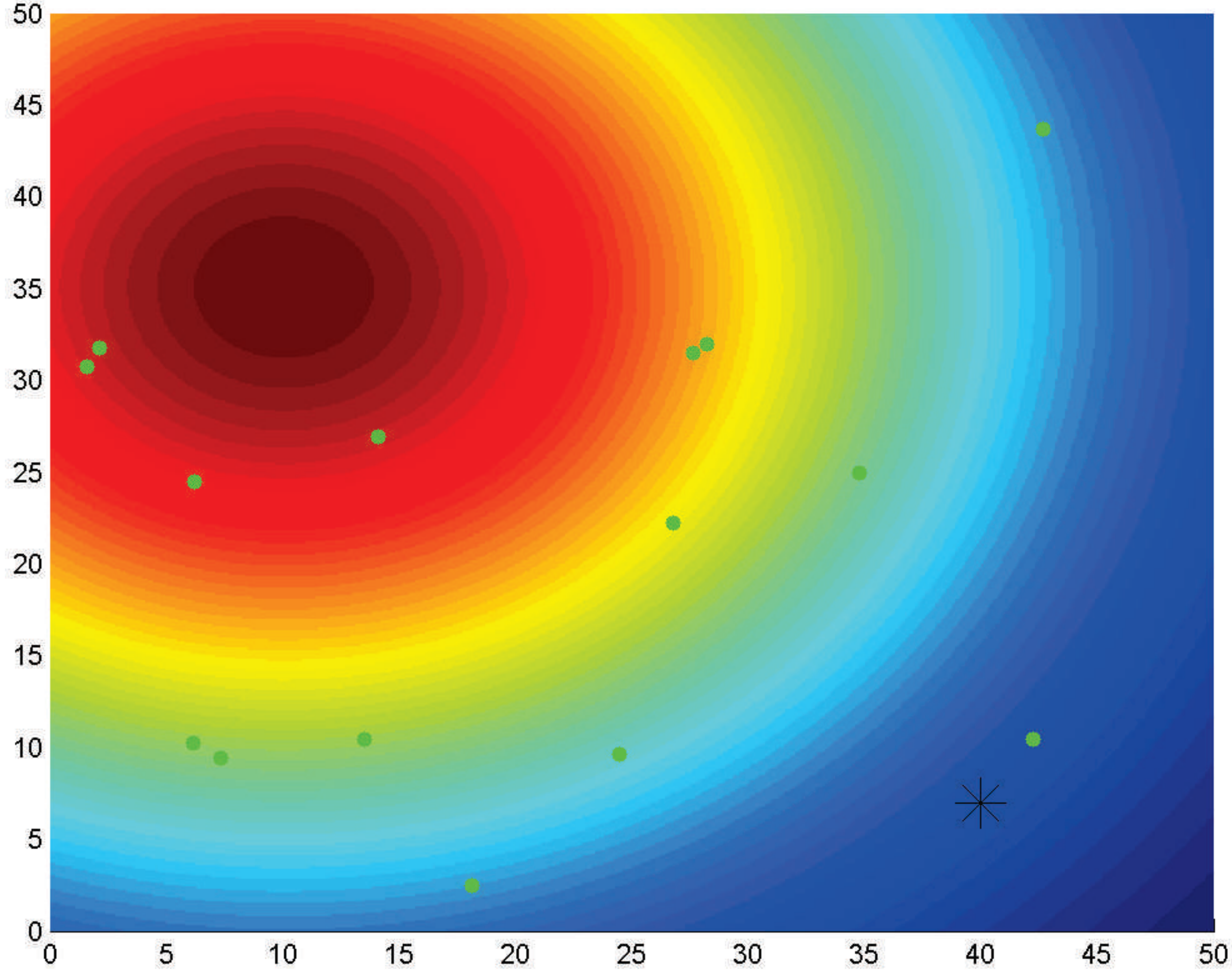
$$k_3^i(\eta_i = 0) > k_3^i(\eta_i = 1)$$

Faulty agents are "pushed" far from the formation center

Reconfiguration



Local approach: complete loop simulation



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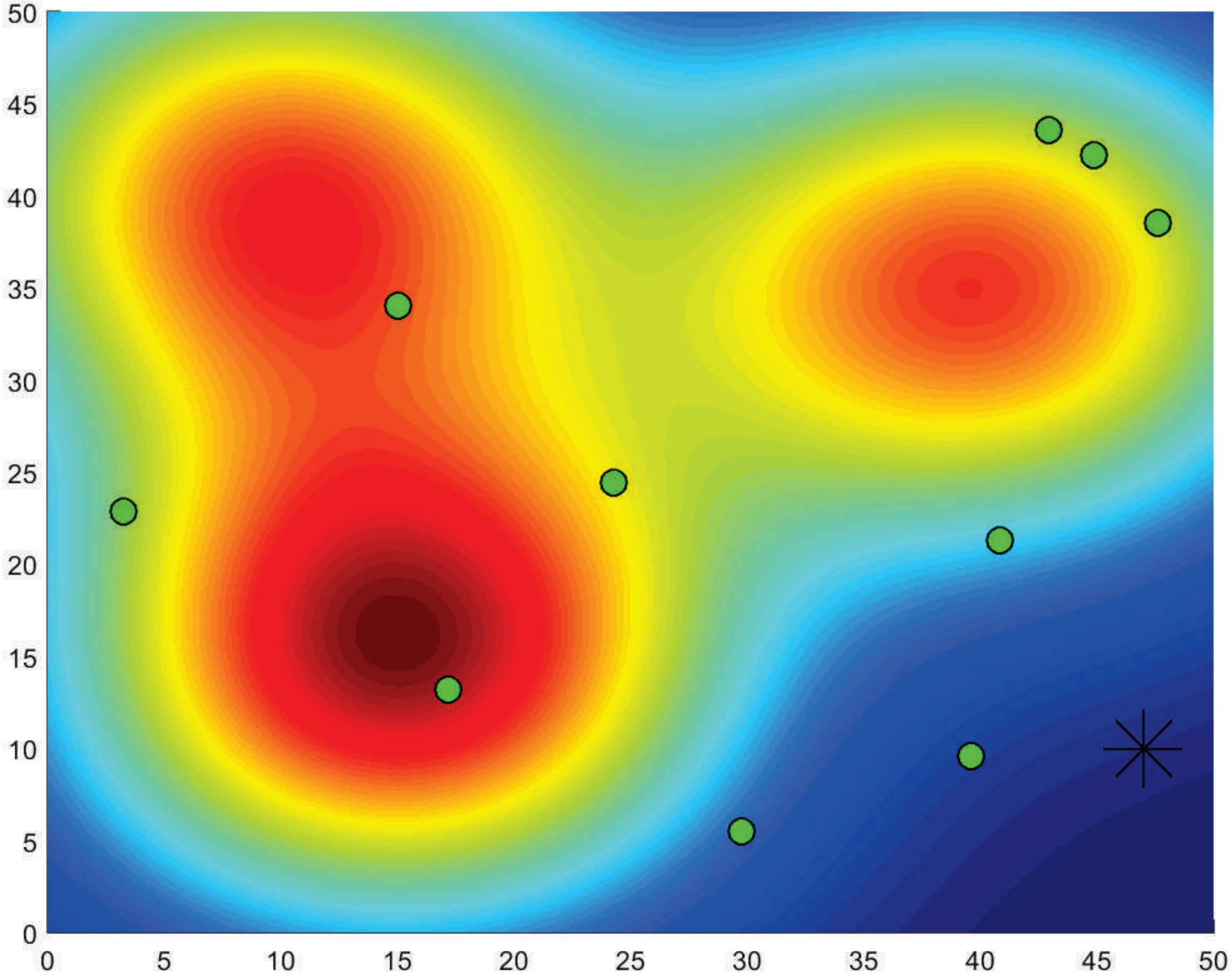
Local approach: complete loop simulation



Section 3

Global Approach

Kriging

Unknown field $\phi(\mathbf{x})$ modeled as

$$Y(\mathbf{x}) = \mathbf{r}(\mathbf{x})^T \beta + Z(\mathbf{x})$$

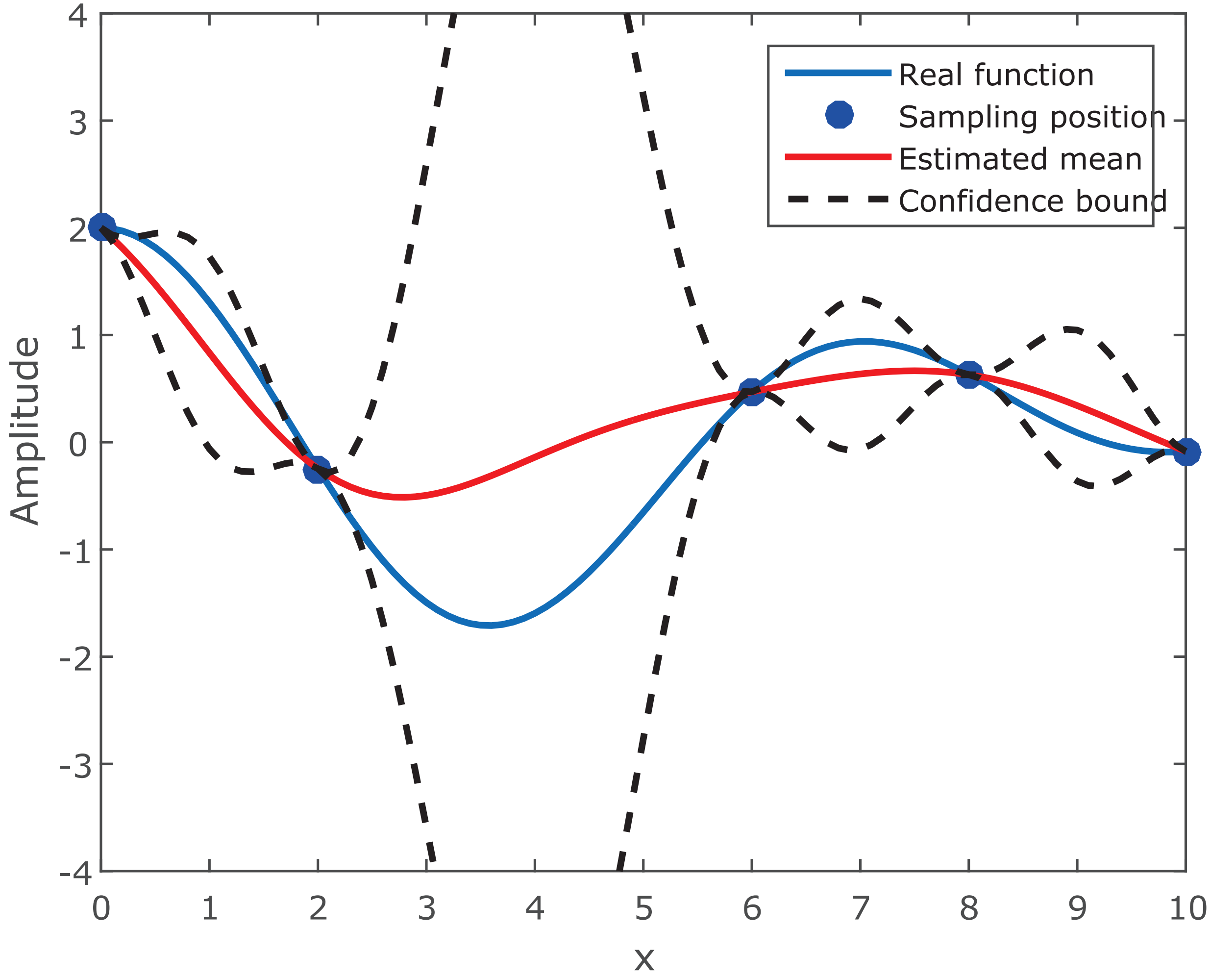
- $\mathbf{r}(\mathbf{x})$ regression vector
- β parameter vector
- $Z(\mathbf{x})$ zero-mean Gaussian process with covariance $C(Z(\mathbf{x}_1), Z(\mathbf{x}_2))$

Kriging provides a Gaussian distribution for each $\mathbf{x}$ with

- a mean value $\mu(\mathbf{x})$
- a prediction variance $\sigma^2(\mathbf{x})$

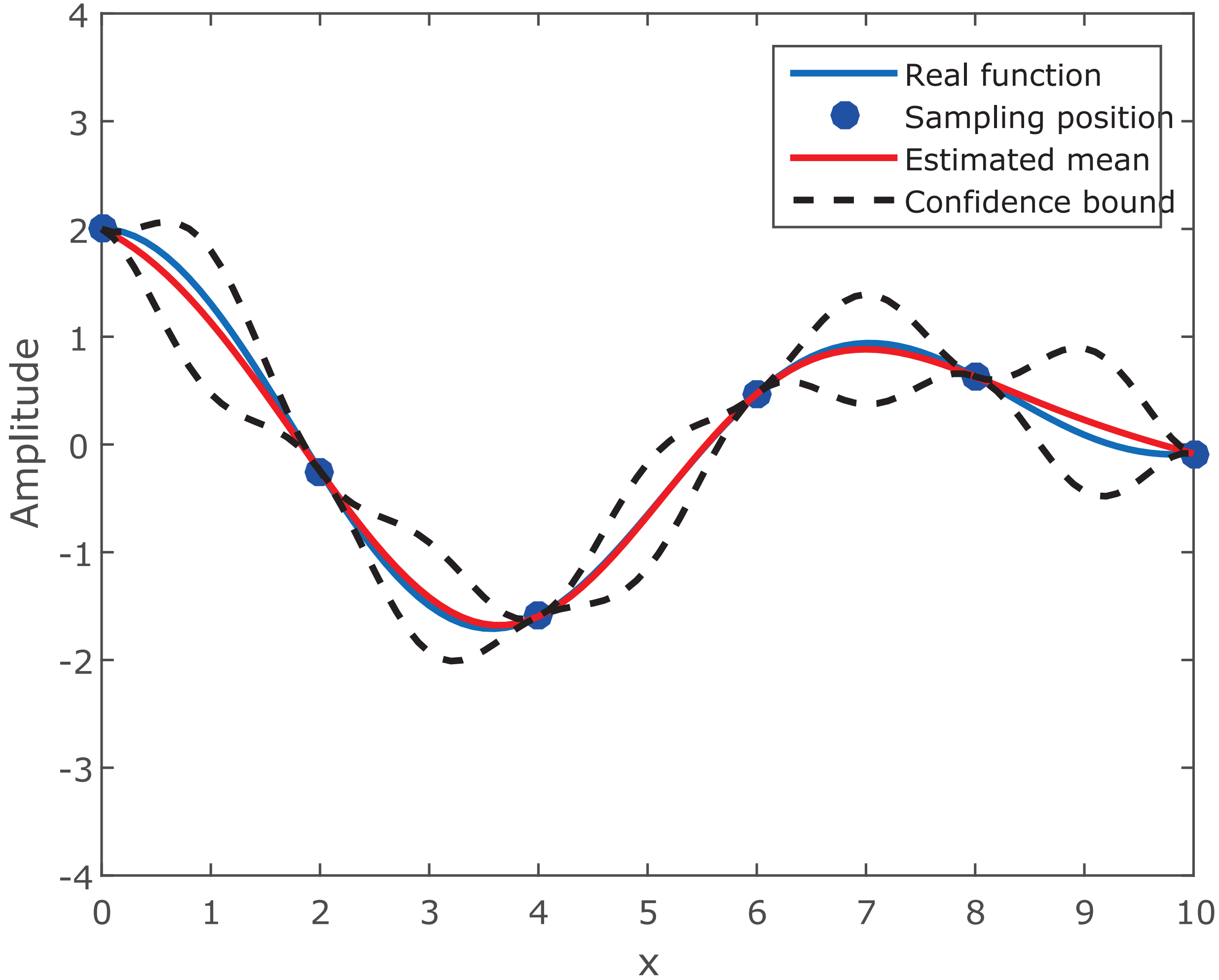
How to choose sampling points ?

Kriging



Real function
Sampling position
Estimated mean
Confidence bound
Amplitude
x
4
3
2
1
0
-1
-2
-3
-4
0 1 2 3 4 5 6 7 8 9 10

Kriging


Real function
Sampling position
Estimated mean
Confidence bound
Amplitude
x
4
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-1
-2
-3
-4
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8
9
10

Kriging-based existing sampling criteria

Kriging-based sampling criterion for seeking $\mathbf{x}_M = \arg\max_{\mathbf{x}\in D} \phi(\mathbf{x})$

- For optimizing costly-to-evaluate functions
- Based on n measurements, choose the $n+1$-th

- ***Kushner, 1962***
- Expected improvement *(Jones, 1998)*
- Confidence bound *(Cox, 1997)*

$$\mathscr{C}_{\text{Kushner}}(\mathbf{x}) = P(\mu(\mathbf{x}) > f_{\max} + \varepsilon)$$

Kriging-based existing sampling criteria

Kriging-based sampling criterion for seeking $\mathbf{x}_M = \arg\max_{\mathbf{x}\in D} \phi(\mathbf{x})$

- ♦ For optimizing costly-to-evaluate functions
- ♦ Based on n measurements, choose the $n+1$-th

- *Kushner, 1962*
- **Expected improvement *(Jones, 1998)***
- Confidence bound *(Cox, 1997)*

$$\mathscr{C}_{\mathrm{EI}}(\mathbf{x}) = (\mu(\mathbf{x}) - f_{\max})\Psi(z) + \hat{\sigma}(\mathbf{x})\psi(z)$$

$$z = \frac{\mu(\mathbf{x}) - f_{\max}}{\hat{\sigma}(\mathbf{x})}$$

Kriging-based existing sampling criteria

Kriging-based sampling criterion for seeking $\mathbf{x}_M = \arg\max_{\mathbf{x}\in D} \phi(\mathbf{x})$

- For optimizing costly-to-evaluate functions
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- *Kushner, 1962*
- Expected improvement *(Jones, 1998)*
- **Confidence bound *(Cox, 1997)***

$$\mathscr{C}_{\mathrm{lcb}}(\mathbf{x}) = \mu(\mathbf{x}) + b_{\mathrm{lcb}}\hat{\sigma}(\mathbf{x})$$

Existing Kriging-based sampling criteria for MVS

Now looking for all vehicle positions $\mathbf{X}$

- ***Choi, 2008***
- *Xu & Choi, 2011*

$$\mathscr{C}_{\text{Choi}}(\mathbf{X}(t)) = \frac{\sum_{p=1}^{4} \lambda_p(t) \Xi_p(\mathbf{X}(t), t)}{\sum_{p=1}^{4} \lambda_p(t)}$$

$\Xi_1 = \mu,\ \Xi_2 = -\mu,\ \Xi_3 = \sigma^2,\ \Xi_4 = \ln(2\pi\sigma^2)$

Minimise uncertainty mean on $\mathscr{I}$, grid of interest points.

- More exploration criteria than global optimization criteria
- Do not take into account vehicle dynamics explicitely

Existing Kriging-based sampling criteria for MVS

Now looking for all vehicle positions **X**

- *Choi, 2008*
- ***Xu & Choi, 2011***

$$\mathscr{C}_{\mathrm{Xu}}(\mathbf{X}(t)) = \frac{1}{|\mathscr{J}|}\sum_{j\in\mathscr{J}} \sigma^2_{z_j}(\mathbf{X}(t))$$

$\Xi_1 = \mu$, $\Xi_2 = -\mu$, $\Xi_3 = \sigma^2$, $\Xi_4 = \ln(2\pi\sigma^2)$

Minimise uncertainy mean on $\mathscr{J}$, grid of interest points.

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Proposed criterion

Goals

- Locate global maximum
- Limit exploration to areas of interest
- Take into account vehicle dynamics

$$J_i^{(k)}(\mathbf{x}) = \|\mathbf{x}_i(t_k) - \mathbf{x}\|^2 - \sum_{j \in \mathcal{N}_i(t_k)} \alpha \|\mathbf{x}_j(t_k) - \mathbf{x}\|^2,$$

$$\mathbf{x}_i^d(t_k) = \arg\min_{\mathbf{x} \in D} \left\{ J_i^{(k)}(\mathbf{x}) \right\}$$

$$\text{s.t. } \hat{\phi}_{i,k}(\mathbf{x}) + b\sigma_{\phi,i,k}(\mathbf{x}) > f^i_{\max}(t_k)$$

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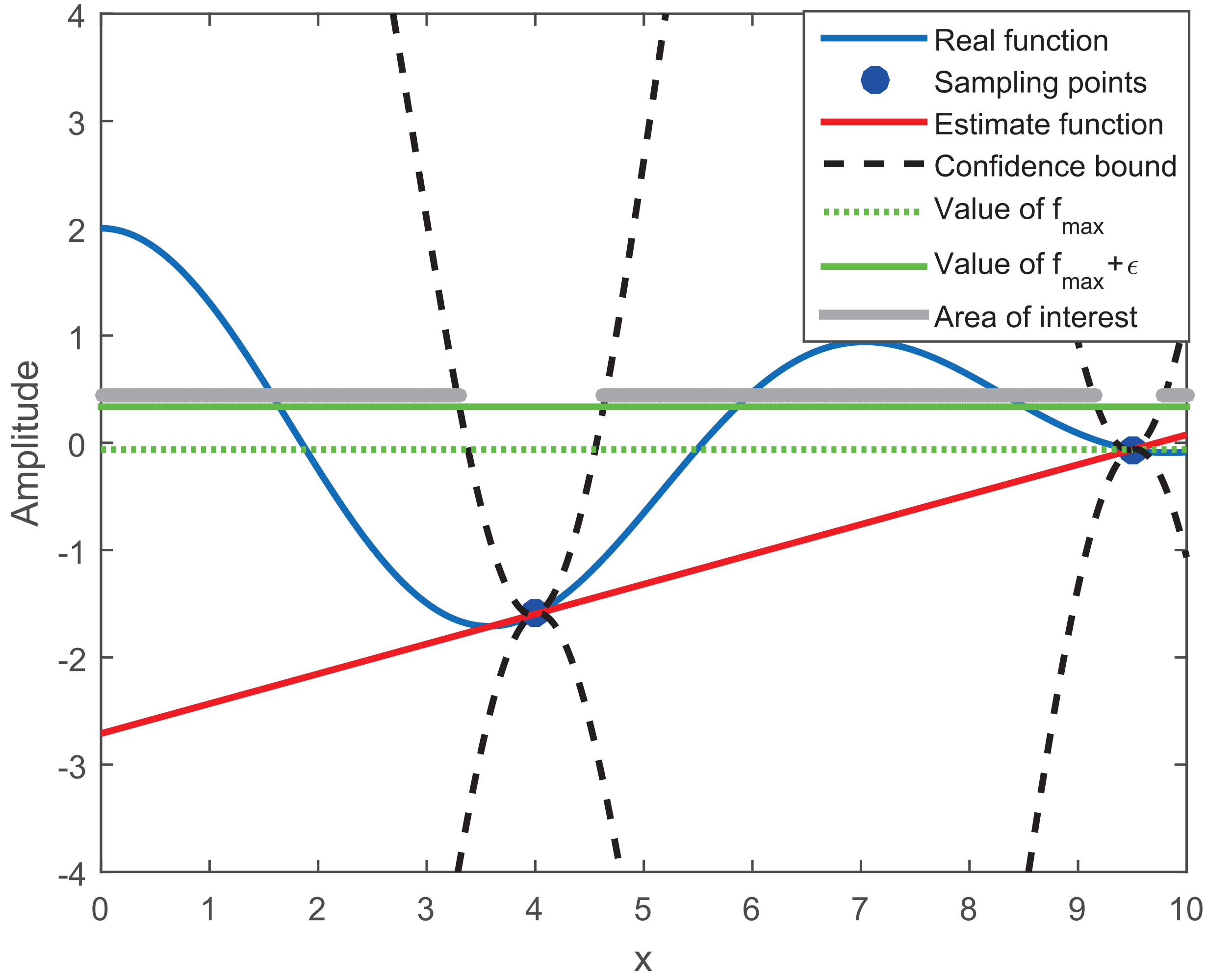
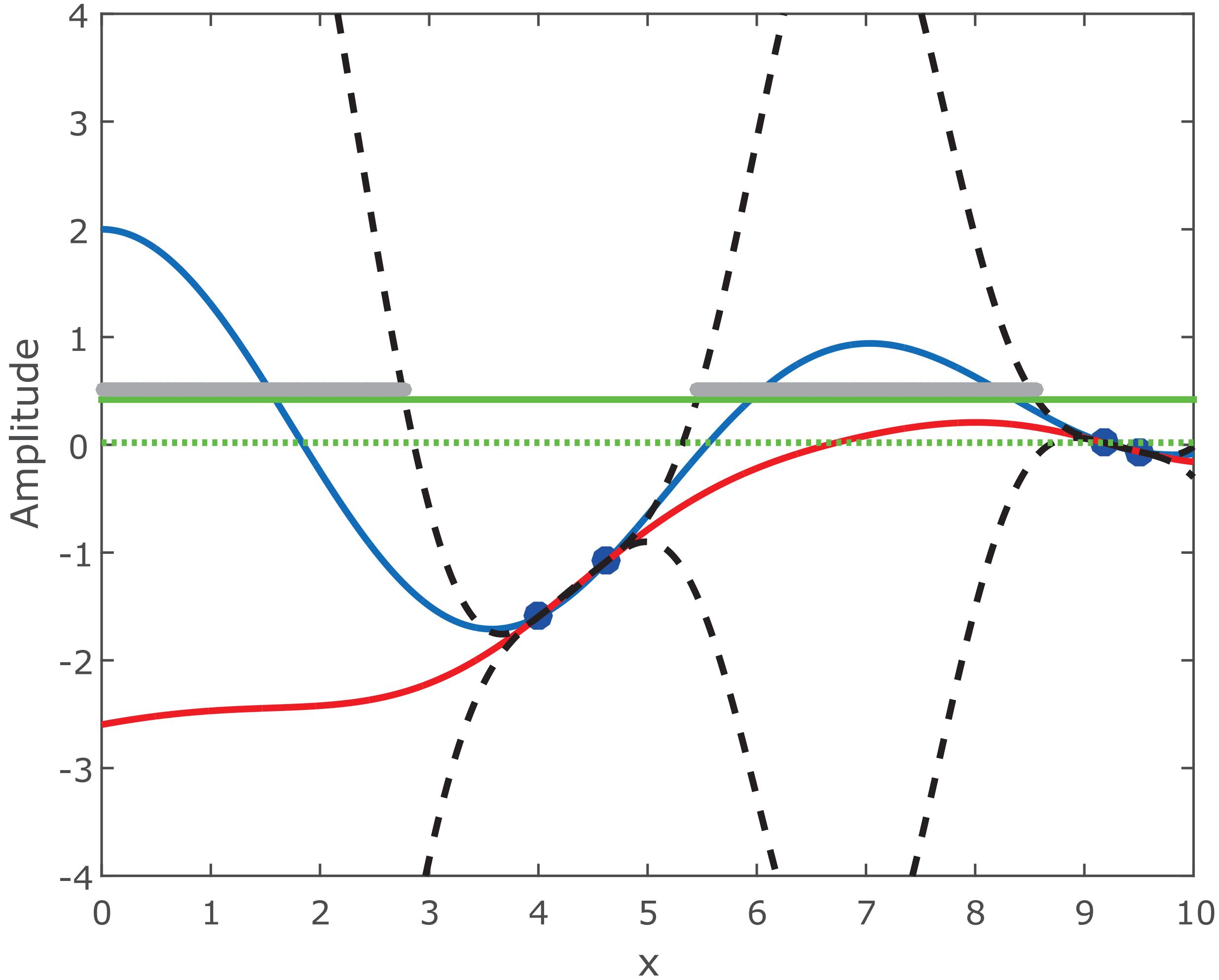
Criterion illustration



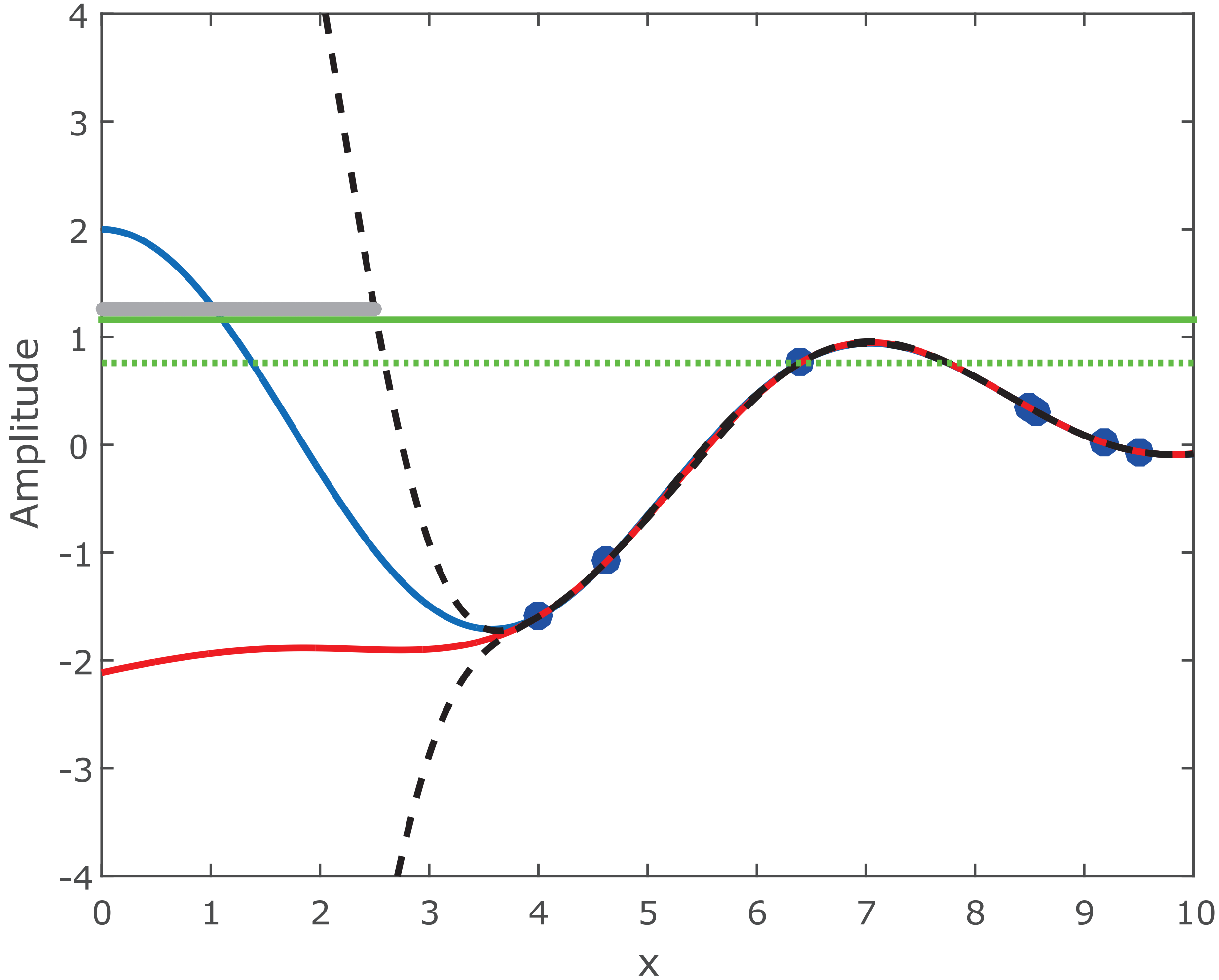
Instant 1

Criterion illustration



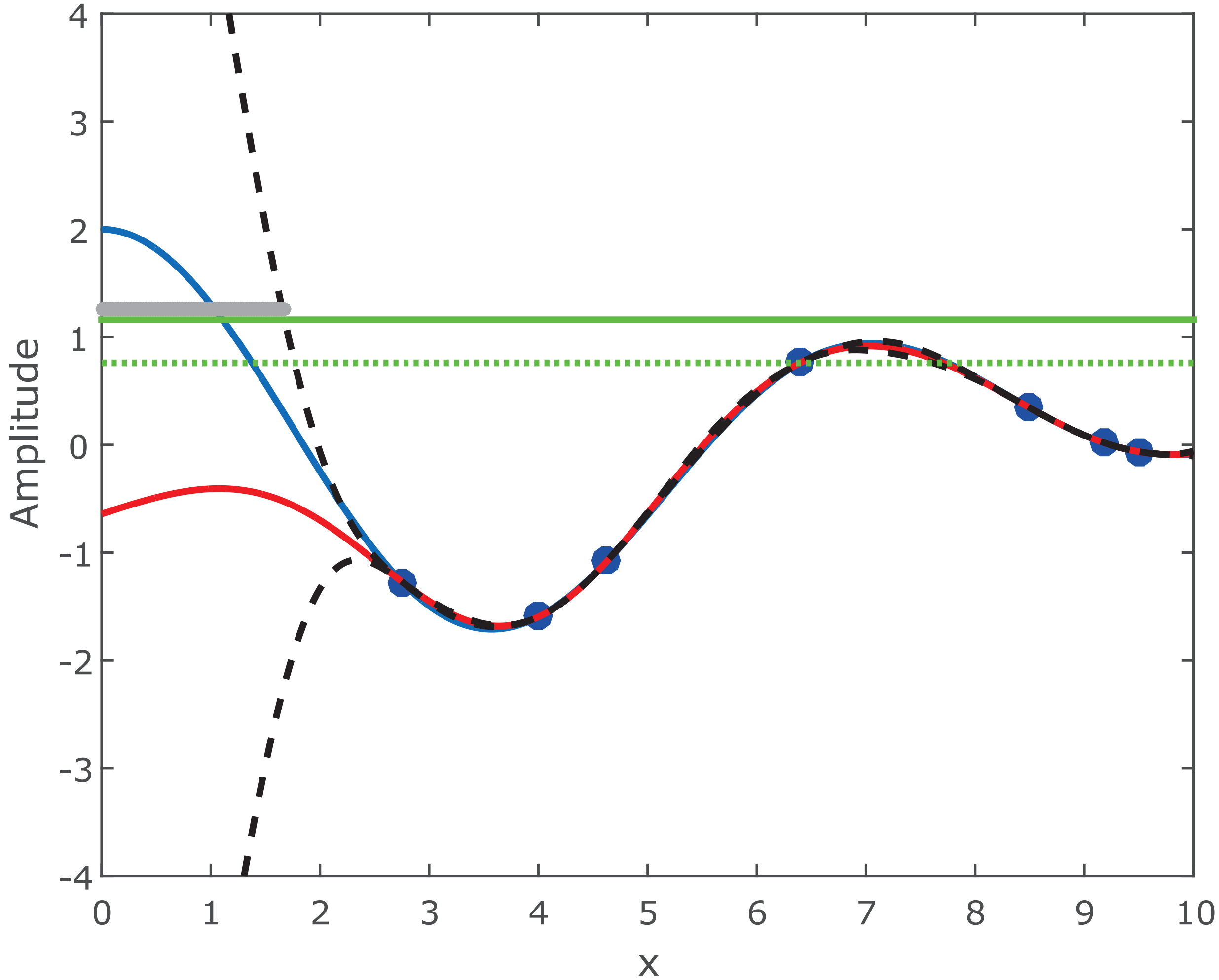
Instant 2

Criterion illustration



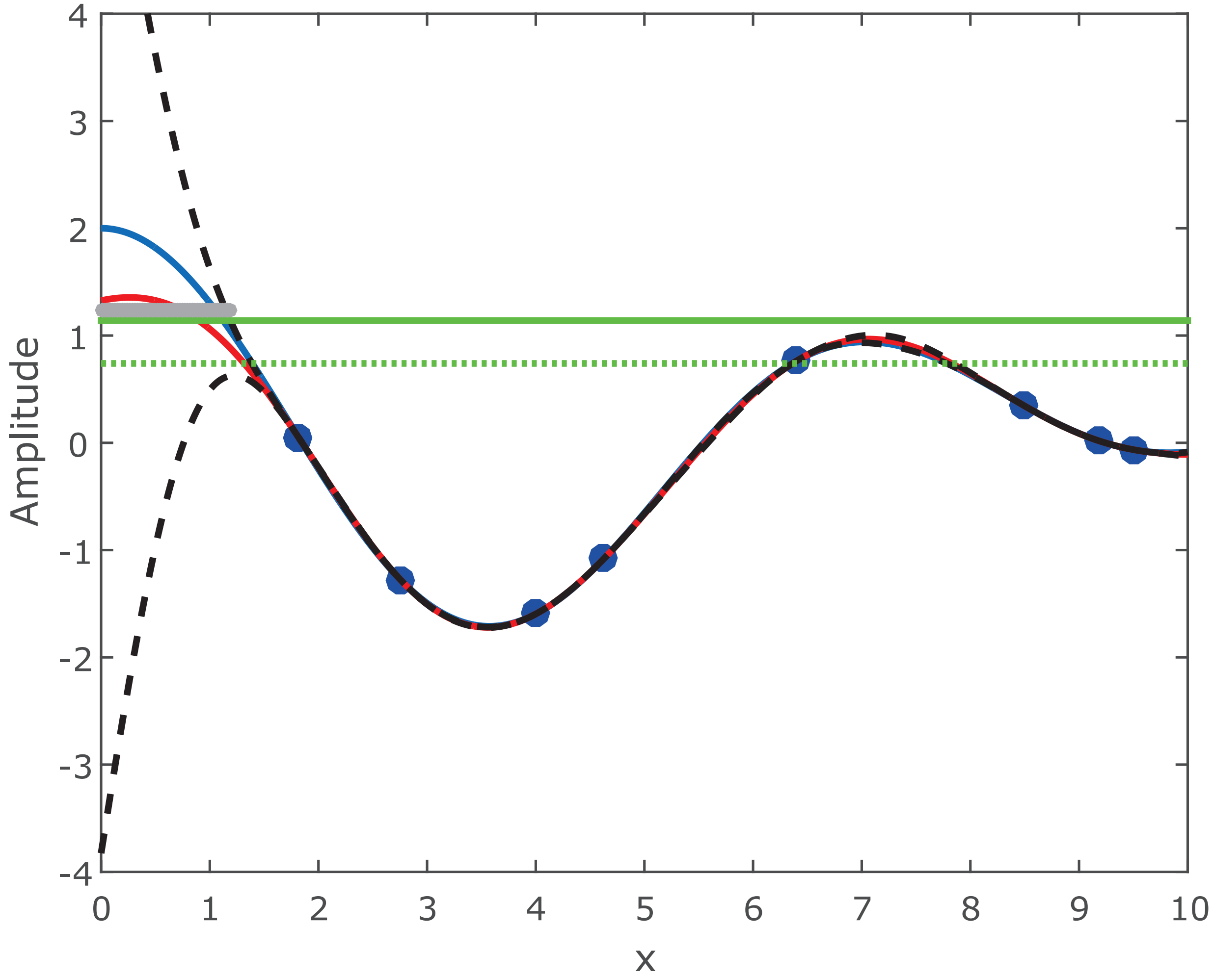
Instant 3

Criterion illustration



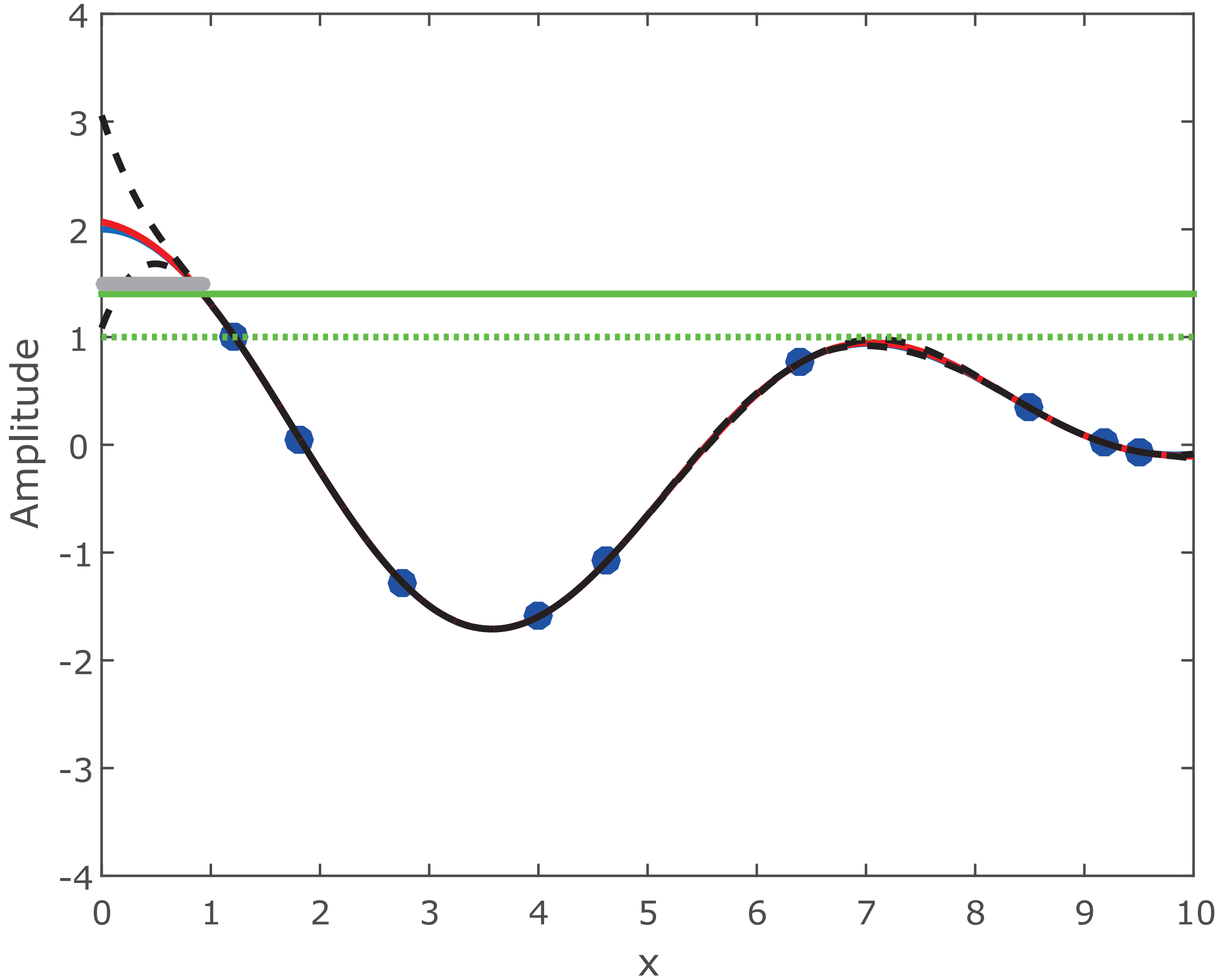
Instant 4

Criterion illustration



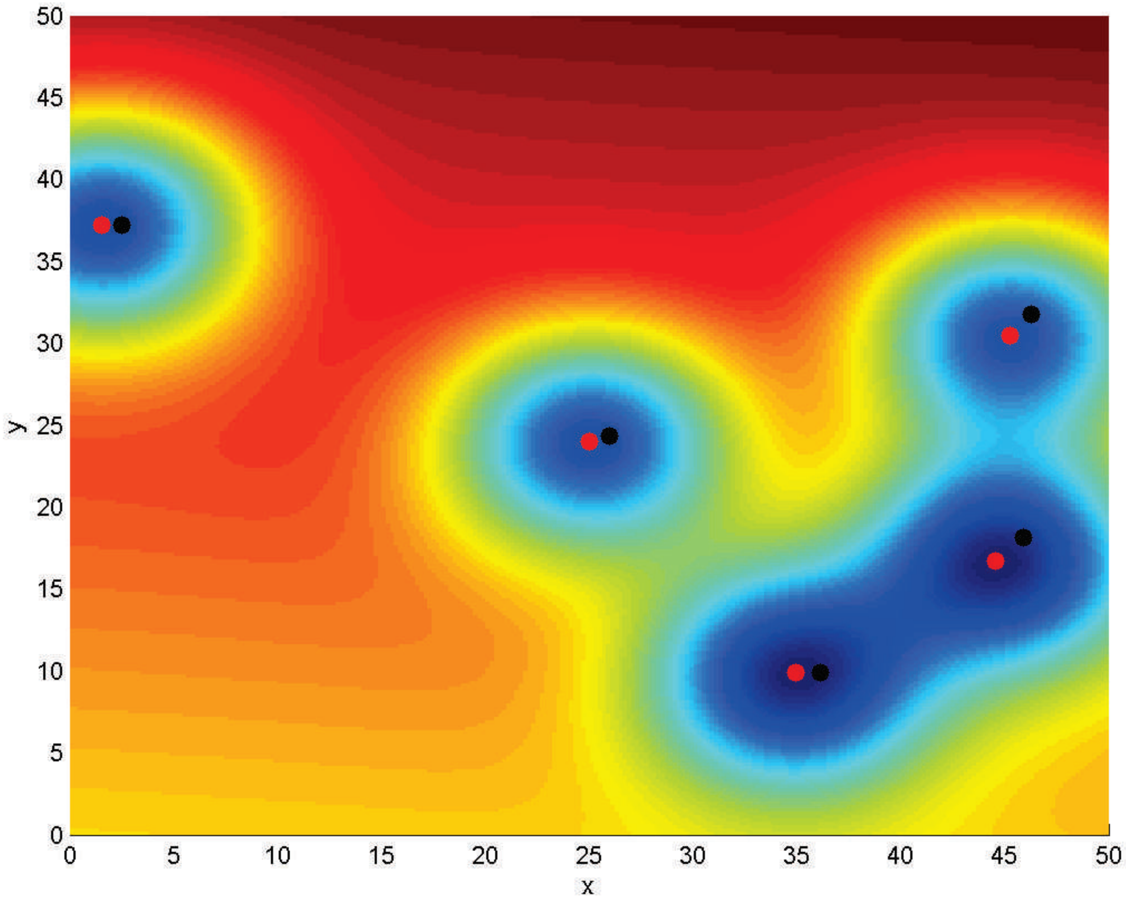
Instant 5

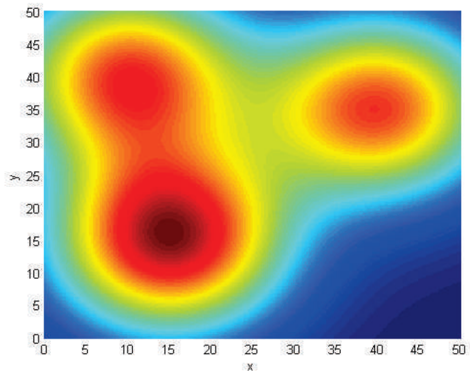
Criterion illustration



Instant 6

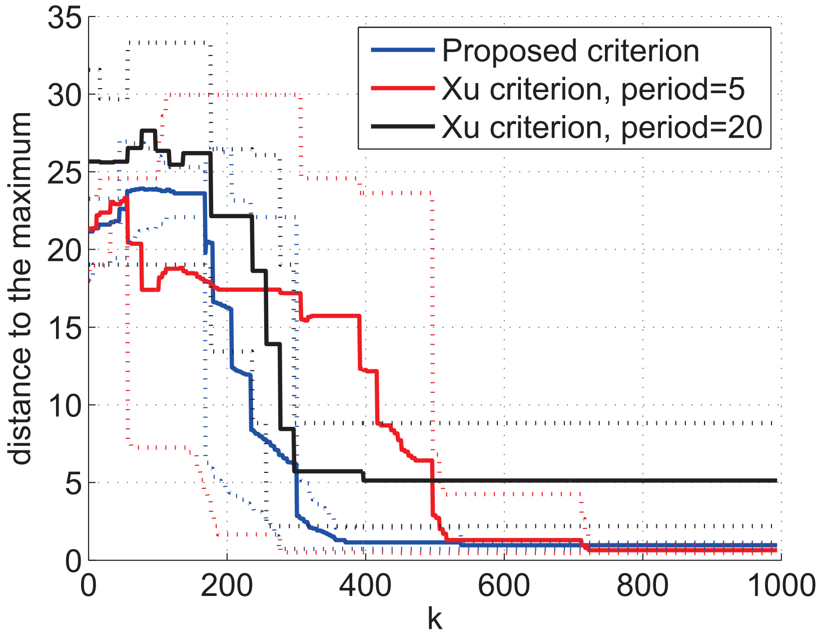
Global approach : full simulation

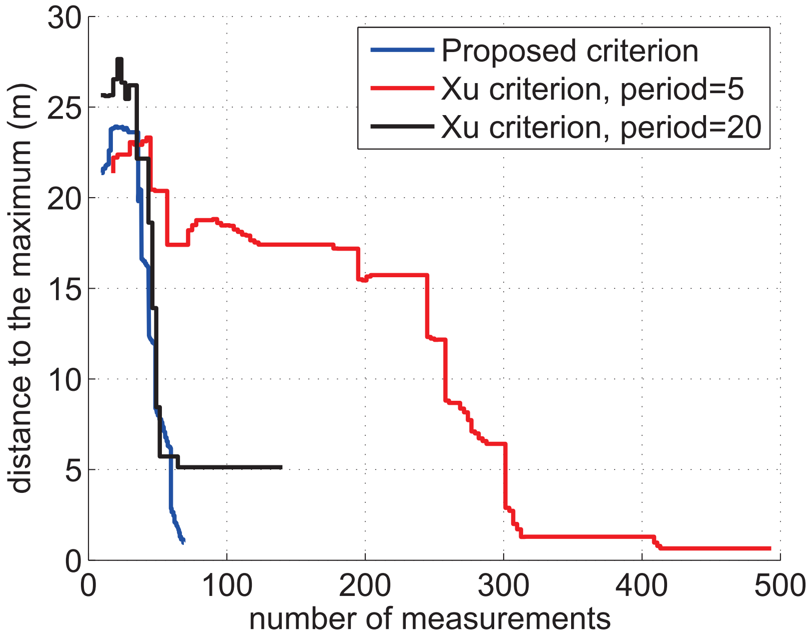




A. Kahn, J. Marzat, H. Piet-Lahanier, M. Kieffer, Global extremum seeking by Kriging with a multi-agent system, 17th IFAC SYSID, Beijing China, 2015

Comparison with reference method





Contribution

- Quick convergence to a small error with limited measurements

Limitations

- True covariance parameters usually unknown

Conclusions and perspectives

Two approaches for maximum location with a multi-vehicle system

Local approach

- Cooperative estimation with associated optimal placement
- Fault detection and identification
- Formation control with reconfiguration

Global approach

- Kriging-based criterion for global optimization to limit search area
- Perspectives
  - Fault diagnosis and reconfiguration with Kriging model
  - Incorporate communication constraints in criterion

Publications

- Kahn, A., Marzat, J., Piet Lahanier, H. *Formation flying control via elliptical virtual structure*, IEEE International Conference on Networking, Sensing and Control, 158-163, Evry, France, (2013)
- Piet Lahanier, H., Kahn, A., Marzat, J. *Cooperative guidance laws for maneuvering target interception*, IFAC Symposium on Automatic Control in Aerospace, 296-301, Würzburg, Germany, (2013)
- Marzat, J., Piet Lahanier, H., Kahn, A. *Cooperative guidance of Lego Mindstorms NXT mobile robots*, 11th International Conference on Informatics in Control, Automation and Robotics, 605-610, Vienne, Austria, (2014)
- Bertrand S., Marzat J., Piet-Lahanier H., Kahn A., Rochefort Y., *MPC Strategies for Cooperative Guidance of Autonomous Vehicles*, Aerospace Lab Journal, 8, 1-18, (2014)
- Kahn, A., Marzat, J., Piet Lahanier, H., Kieffer, M. *Cooperative estimation with outlier detection and fleet reconfiguration for multi-agent systems*, IFAC Workshop on Multi-Vehicule Systems, 11-16, Genova, Italy, (2015)
- Kahn, A., Marzat, J., Piet Lahanier, H., Kieffer, M. *Global extremum seeking by Kriging with a multi-agent system*, 17th IFAC Symposium on System Identification, Bejing, China (2015)